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# Assignment 1: Question 1

**Acknowledgments.** This question was completed only with the teachings from CS courses here at uWaterloo.

## Asymptotics

Prove or disprove each of the following statements.

(a) For any constant $b > 0$, the function $f : n \mapsto 1 + b + b^2 + b^3 + \cdots + b^n$ satisfies

$$f(n) = \begin{cases} \Theta(b^n) & \text{if } b > 1 \\ \Theta(1) & \text{if } b \leq 1. \end{cases}$$

**Solution.** Since f(n) is a geometric series, we can express it as $\sum_{i=1}^{n} b^i = \frac{1-b^N}{1-b} = \frac{b^n-1}{b-1}$ Using this:
When b=1: $\lim n \to \infty \frac{\frac{b^n-1}{b-1}}{b^n} = \frac{b^n-1}{b^{n+1}-b}$, which, by :'Hopital's rule $= 1 \div b$. Since $01 \div b\infty$ , we have a $\theta bound.$

(b) For every pair of functions $f, g : \mathbb{Z}^+ \to \mathbb{R}^{\geq 1}$ that satisfy $f = \Theta(g)$, the functions $F : n \mapsto 2^{f(n)}$ and $G : n \mapsto 2^{g(n)}$ also satisfy $F = \Theta(G)$.

**Solution.** (ENTER YOUR SOLUTION HERE.)

(c) For every pair of functions $f, g : \mathbb{Z}^+ \to \mathbb{R}^{\geq 1}$ that satisfy $f = o(g)$, the functions $F : n \mapsto 2^{f(n)}$ and $G : n \mapsto 2^{g(n)}$ also satisfy $F = o(G)$.

**Solution.** (ENTER YOUR SOLUTION HERE.)